03/2/26

I. Give an example of a Pseudo metric space.

Soln. Let us consider the set R and $d : R \times R \to R$ such that

$$d(x,y) = |x^2 - y^2| \ \forall\, x, y \in R.$$

one can easily verify that

(i) $d(x,y) \geq 0 \ \forall\, x, y \in R$

(iii) $d(x,y) = |x^2 - y^2| = |y^2 - x^2| = d(y,x)$

(iv) $d(x,y) \leq d(x,z) + d(z,y) \ \forall\, x, y, z \in X.$

Now, we verify whether

$$d(x,y) = 0 \Leftrightarrow x = y \ \forall\, x, y \in R$$

Let $d(x,y) = 0$

$\Rightarrow |x^2 - y^2| = 0$

$\Rightarrow x^2 - y^2 = 0 \Rightarrow x^2 = y^2$

$\Rightarrow x = \pm y$

$\therefore d(x,y) = 0 \Rightarrow x = y$ or $x = -y$.

So, $d(x,y) = 0$ doesnot imply $x = y$.

Hence ~~the set~~ (R, d) is a Pseudo-metric space.